

**B.Tech I Year (RR) Supplementary Examinations, December 2010**  
**MATHEMATICS-I**  
 (Common to all branches)

Time: 3 hours

Max Marks: 80

Answer any FIVE questions  
 All questions carry equal marks

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1. (a) Test the following series for convergence or divergence.  

$$\frac{1}{2} + \frac{\sqrt{2}}{3} + \frac{\sqrt{3}}{8} + \dots + \frac{\sqrt{n}}{n^2-1}$$
 (b) Test whether the following series is absolutely convergent.  

$$\sum_1^{\infty} (-1)^{n+1} (\sqrt{n+1} - \sqrt{n})$$
 (c) Verify Lagrange's mean value theorem  $f(x) = \log_e x$  in  $[1, e]$ .
2. (a) If  $u = \log(x^2+y^2) + \tan^{-1}(y/x)$  prove that  $u_{xx} + u_{yy} = 0$ .  
 (b) Define curvature, center of curvature, radius of curvature and circle of curvature.
3. (a) Trace the curve  $9ay^2 = x(x-3a)^2$ .  
 (b) Prove that the length of the arc of the parabola  $y^2 = 4ax$  cut off by its latus rectum is  $2a[\sqrt{2} + \log(1 + \sqrt{2})]$ .
4. (a) Form the differential equation by eliminating the arbitrary constant :  $\log y/x = cx$ .  
 (b) Solve the differential equation:  $dr + (2r \cot\theta + \sin 2\theta) d\theta = 0$ .  
 (c) The number N of bacteria in a culture groups at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after  $1 \frac{1}{2}$  hour.
5. (a) Solve the differential equation:  $(D^2 + 4)y = \cos x$ .  
 (b) Solve the differential equation:  $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + 4y = (1+x)^2$
6. (a) Solve the differential equation  $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 12x = e^{3t}$  given that  $x(0) = 1$  and  $x'(0) = -2$  using laplace transforms.  
 (b) Evaluate by transforming in to polar co ordinates  

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} y \sqrt{x^2 + y^2} dx dy$$
7. (a) Find  $\text{curl}[\mathbf{r}f(r)]$  where  $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ ,  $r = |\mathbf{r}|$ .  
 (b) Find the work done in moving a particle in the force field  
 $\mathbf{F} = 3x^2\mathbf{i} + (2xz - y)\mathbf{j} + z\mathbf{k}$  along the straight line from  $(0, 0, 0)$  to  $(2, 1, 3)$ .
8. Verify Green's theorem for  $\oint_C [(3x - 8y^2)dx + (4y - 6xy)dy]$  where C is the region bounded by  $x=0, y=0$  and  $x + y = 1$ .

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