Code No: R5210201 **R5**

II B.Tech I Semester(R05) Supplementary Examinations, December 2009

MATHEMATICS-III

(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Computer Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions

All Questions carry equal marks

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1. (a) Define Gamma function and evaluate $\Gamma(1/2)$.
   (b) Show that $\Gamma(1/2)\Gamma(2n) = 2^{2n-1}\ \Gamma(n)\ \Gamma(n+1/2)$.
   (c) Define Beta function and show that $\beta(m,n) = \beta(n,m)$. [6+6+4]

2. (a) Prove that $\frac{1}{\sqrt{1-2tx+t^2}} = P_0(x) + P_1(x)\ t + P_2(x)\ t^2 + ...$
   (b) Write $J_{5/2}(x)$ in finite form. [8+8]

3. (a) Show that the real and imaginary parts of an analytic function $f(z) = u(r,\theta) + i\ v(r,\theta)$ satisfy the Laplace equation in polar form $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$ and $\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} = 0$ respectively.
   (b) If u is a harmonic function, show that $w = u^2$ is not a harmonic function unless u is a constant. [8+8]

4. (a) Evaluate $\int_C \frac{Cos\, z - \sin\, z\ \ dz}{(z+\pi/2)^3}$ with C: $|z| = 2$ using Cauchy's integral formula.
   (b) Evaluate $\int_{1-i}^{2+i} (2x + 1 + iy)dz$ along (1-i) to (2+i) using Cauchy's integral formula. [8+8]

5. (a) State and prove Taylor's theorem.
   (b) Find the Laurent series expansion of the function $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$. [8+8]

6. (a) State and prove Residue theorem.
   (b) Evaluate $\int_C \tan z dz$ where C is the circle $|z| = 2$ by using residue theorem.
   (c) Evaluate $\int_C \frac{3\cos z}{(\pi+3z)}$ where C is the unit circle by using residue theorem. [5+6+5]

7. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)^2}$, $a>0$, $b>0$ using residue theorem.
   (b) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+1)^3}$ using residue theorem. [8+8]

8. (a) Show that the transformation $w = i(1-z) / (i-z)$, maps the interior of the circle $|z|=1$ in to the upper half of the w-plane, the upper semi circle into positive half of real axis and lower semi circle into negative half of the real axis.
   (b) By the transformation $w = z^2$ show that the circle $|z-a| = c$ (a and c are real) in the z plane correspond to the limacons in the w-plane. [8+8]

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