

I B.Tech Supplementary Examinations, January/February 2010
MATHEMATICS-I

(Common to Civil Engineering, Electrical & Electronics Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Electronics & Instrumentation Engineering, Information Technology, Electronics & Control Engineering, Computer Science & Systems Engineering, Electronics & Computer Engineering, Bio Technology)

Time: 3 hours

Max Marks: 80

Answer any FIVE questions
All questions carry equal marks

1. (a) Form the differential equation by eliminating the arbitrary constant, $\sin^{-1} x + \sin^{-1} y = c$.
(b) An object whose temperature is $75^{\circ}C$ cools in an atmosphere of constant temperature $25^{\circ}C$ at the rate of $k\theta$, θ being the excess temperature of the body over that of the atmosphere. If after 10 minutes, the temperature of the object falls to $65^{\circ}C$, find its temperature after 20 minutes. Also find the time required to cool down to $55^{\circ}C$.
2. (a) Solve: $(D^2 - 3D + 4)y = 0$.
(b) Solve: $y'' - y' - 2y = 3e^{2x}$ given that $y(0) = 0, y'(0) = -2$.
3. (a) Verify Lagrange's mean value theorem for $f(x) = \log x$ in $[1, e]$.
(b) If $x=v+w, y=w+u, z=u+v$, then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = 2$.
4. (a) Find the radius of curvature at the point $(\frac{3a}{2}, \frac{3a}{2})$ of the curve $x^3 + y^3 = 3axy$.
(b) Trace the curve: $y^2(a+x) = x^2(3a-x)$.
5. (a) Find the length of the arc of the curve $x = e^{\theta} \sin \theta, y = e^{\theta} \cos \theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$.
(b) Evaluate: $\int_0^5 \int_0^{x^2} x(x^2 + y^2) dx dy$.
6. (a) Test for convergence of the series:
$$\sum_{n=1}^{\infty} [\sqrt{n^4 + 1} - \sqrt{n^4 - 1}].$$

(b) Test the convergence of the series:
$$\sum_{n=1}^{\infty} \frac{x^{2n}}{(n+1)\sqrt{n}}.$$
7. (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.
(b) Use divergence theorem to evaluate:
 $\int \int_S F \cdot ds$ where $F = x^3 i + y^3 j + z^3 k$ and 'S' is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.
8. (a) Find the Laplace transform of the function:
i. $e^{at} \cosh bt$.
ii. $t^2 \cdot e^{-2t}$.
(b) Find the inverse transform of the following functions:
i. $\frac{1}{s^2(s^2+a^2)}$.
ii. $\frac{1}{s(s+1)(s+2)}$.
